Reducing the Effects of Correlation Between Multiple Clocks on Clock Ensemble Time Scale

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***Summary*—This paper proposed a new weighting method for clock ensemble: redundant-subspace-weighting (RSW). In this approach, multiple overlapping subspaces are constructed in clock ensemble. By superimposition averaging of different subspace covariances, RSW could reduce the correlations between clocks located in the same laboratory. Preliminary simulation results show its potential capability to enhance the frequency stability of clock ensemble time scale.**

***Keywords*—weighting method; redundant subspace; clock ensemble; time scale; atomic clock**

I. Introduction

Precision time and frequency technology are required in various fields, including satellite navigation, communication engineering, and geographic information science. Time scales always are maintained by the ensembles of atomic clocks that clocks are independent of each other.

But the clocks in the similar environments is hard to avoid the similar fluctuations. For the output of atomic clock is easily effected by the environment of the laboratory, such as temperature, humidity, and magnetic fields[1]. With the growing demand of the high-precision time services, the atomic clock correlation in an ensemble has gotten the attention of the researchers. Using a high-performance atomic clock as a reference or remote comparison using optical-fiber links are the traditional methods to eliminate the effect of the clock correlation. Therefore, a low-cost and maneuverable method is needed to reduce the influence of clock group correlation on time scale.

II. Method

In clock ensemble analyst, the weight-and-sum (WAS) method[2] is a simple yet powerful technique. The general definition of a WAS for the ensemble time scale is

$$z(t)=\sum_{i=0}^{M-1} w_i(t)_i x_i(t) \tag{1}$$

where z(t) is the output of the clock ensemble time scale, and w(t) and x(t) are the weight and residual relative to the reference of the clock, respectively. The weights are used to control the rejection of off-axis interference. Equation (1) can be written as (2) in matrix form

$$z(t)=\boldsymbol{w}(t)^T\boldsymbol{X}(t) \tag{2}$$

The minimum variance weighting (MVW) [3] approach is used here, to find the weights $\boldsymbol{w}$ that minimize the variance of $z(t)$ in (2). The variance of the WAS output for the clock ensemble time scale in (2) can be expressed as

$$E\{|z(t)|^2\} = E\{|\,\boldsymbol{w}(t)^T\boldsymbol{X}(t)\,|^2\} = \boldsymbol{w}(t)^T\boldsymbol{R}(t)\boldsymbol{w}(t) \tag{3}$$

where $\mathrm{R} = \mathrm{E}\{\boldsymbol{X}(t)\boldsymbol{X}(t)^T\}$ is the spatial covariance matrix. The solution of minimum variance optimization can be formulated as

$$\boldsymbol{w}(\mathrm{t}) = \frac{\boldsymbol{R}(t)^{-1}\alpha}{\alpha^T\boldsymbol{R}(t)^{-1}\alpha} \tag{4}$$

where α is a unit vector, α= $[1,1, \cdots 1]^T$, that represents the distribution of all clocks in the clock ensemble. And $\boldsymbol{R}(\mathrm{t})^{-1}$ is the inverse matrix of $\boldsymbol{R}(\mathrm{t})$. If there are no correlations between the measurements, $\boldsymbol{R}(\mathrm{t})$ is a simple diagonal matrix, and it's the key to resolving the correlations. The aim is to modify $\boldsymbol{R}(\mathrm{t})$ to make it more like a diagonal matrix, where the correlations between atomic clocks could be effectively suppressed or eliminated.

In this work, a redundant subspace weighting (RSW) procedure is the proposed to use overlapping clock subsets to get multiple smaller-sized covariance matrices. The RSW can decrease the possibility of negative values in $\boldsymbol{R}(\mathrm{t})$ for the ensemble with the greater number of clocks. In this paper, the true covariance matrix is replaced with an estimate of the average form

$$\widehat{\boldsymbol{R}}[n] = \frac{1}{N}\sum_l \boldsymbol{X}_l[n]\boldsymbol{X}_l^T[n], \tag{5}$$

where N is the number of clock subsets. Each subset represents a subspace of the original residual vector space and $\boldsymbol{X}_l[n]$ is the lth subset. $\boldsymbol{X}_l[n]$ is given by

$$\boldsymbol{X}_l[n] = \begin{bmatrix} \boldsymbol{x}_l[n] \\ \boldsymbol{x}_{l+1}[n] \\ \vdots \\ \boldsymbol{x}_{l+L-1}[n] \end{bmatrix}, \tag{6}$$

where L is the size of a subset. The window size is 2K + 1 temporal samples for each subspace. A sliding window is applied to all clocks and overlapping subsets define a series of redundant subspace. Each subspace is a $L \times Q$ samples matrix that being used to calculate a $L \times L$ covariance matrix.

The general estimation of the covariance matrix with all clock subsets is the mean of all covariance matrices as follow

$$\widehat{\boldsymbol{R}}[n] = \frac{1}{Q(M-L+1)}\sum_{k=-K}^{K}\sum_{l=0}^{M-L}\boldsymbol{X}_l[n-k]\boldsymbol{X}_l^T[n-k], \tag{7}$$

and

$$\bar{\boldsymbol{X}}_l[n] = \left[\bar{\boldsymbol{x}}_l[n] \cdots \bar{\boldsymbol{x}}_{l+L-1}[n]\right]^T. \tag{8}$$

Here, $\bar{\boldsymbol{x}}_l[n] = \boldsymbol{x}_l[n]$, and using $\boldsymbol{R}[n] = \widehat{\boldsymbol{R}}[n]$ in (2), the estimate for the total set as the average over the subsets is

$$\hat{z}[n] = \frac{1}{M-L+1}\sum_{l=0}^{M-L}\boldsymbol{w}[\boldsymbol{n}]^T\bar{\boldsymbol{X}}_l[\boldsymbol{n}]. \tag{9}$$

In diagonal loading, a small term is added to the diagonal of the covariance matrix in (7) before inversion, $\widehat{\boldsymbol{R}}[n]$ is replaced with $\widehat{\boldsymbol{R}}[n] + \varepsilon \boldsymbol{I}$. $\widehat{\boldsymbol{R}}[n]$ then becomes a positive definite matrix. For the stability of the numerical calculation, it is seen empirically that ε in the diagonal loading is proportional to the trace of $\widehat{\boldsymbol{R}}[n]$ and given by

$$\varepsilon = \frac{1}{L} tr\{\widehat{\boldsymbol{R}}[n]\}, \tag{10}$$

where L is the dimension of $\widehat{\boldsymbol{R}}[n]$. Consequently, ε is proportional to the average variance of the clock subset residuals.

III. Simulation

This work used the simulation model of hydrogen maser (HM) mentioned in [1]. Its main simulation parameters are shown as follows: initial phase time of each clock is 0; initial fractional frequency of each clock is 1.0×10^{-14}; frequency drift is 0; diffusion coefficients of frequency modulation noise is 8.8×10^{-14} ; diffusion coefficients of random walk frequency modulation noise is 5.6×10^{-18}; static temperature is -5×10^{-15} /°C; dynamic temperature is -1×10^{-14} /(°C · s^{-1}); magnetic field is 8×10^{-16} /μT; relative humidity is 2×10^{-16} /%.

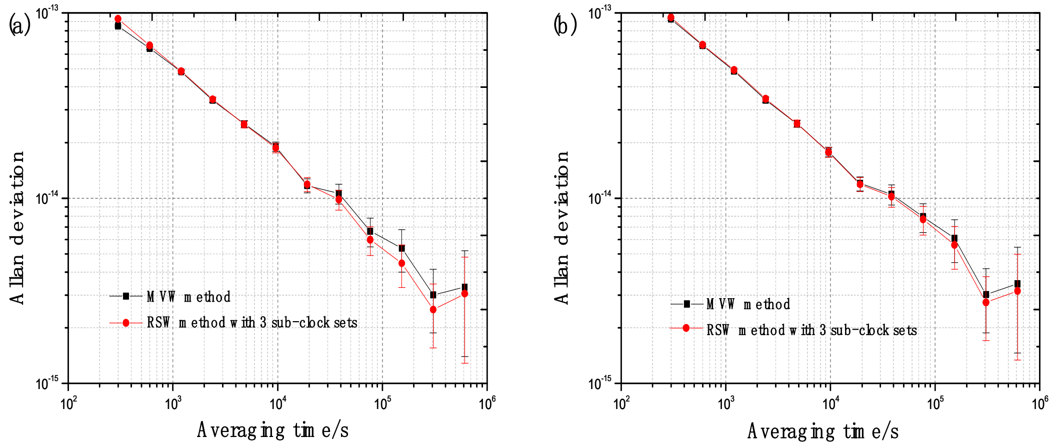

Fig. 1. The Allan deviation of weighted HM-1 to HM-5, relative to an uncorrelated reference HM-6, for different lengths of the sliding window. RSW with N=3 clock subsets. (a) Window size Q=55 min (11 sample points). (b) Window size Q=305 min (61 sample points).

IV. Results and Discussion

A small ensemble of six HMs was established. HM-1 to HM-5 were randomly chosen from group that clocks have correlations. HM-6 was randomly chosen from group that clocks are independent. Using HM-1 to HM-5 and taking the uncorrelated clock HM-6 as a reference, the proposed technique with N=3 clock subsets outperform the MVW method when taking the uncorrelated clock as a reference and when $\tau \geq 3.8 \times 10^4$ s, as shown in Fig.1.

The average performance improvement for different combinations of window sizes was calculated and compared. The average performance improvement (API) is given by

$$API = \frac{1}{T}\sum_t^T \frac{adev_{MVW}(\tau_t) - adev_{RSW}(\tau_t)}{adev_{MVW}(\tau_t)}, \tag{11}$$

where $adev_{\mathrm{MVW}}(\tau_t)$ and $adev_{\mathrm{RSW}}(\tau_t)$ are the Allan deviations of the classic inverse variance weighted method and the proposed technique, respectively, when $\tau = \tau_t$. This improvement was calculated for two different window sizes, Q=55 min and Q=305 min.

During multiple experiments, when the window size Q decreased, there tended to be a higher average performance improvement and standard deviation of this improvement. The performance improvement was also greater when taking an uncorrelated clock as the reference instead of a correlated clock. The average performance improvement reached 3.66% and 2.25% for an uncorrelated reference clock for the same window sizes.

V. Conclusions

The RSW was shown to partially reduce the impact of the correlations between atomic clocks induced by the local environment on the frequency stability of the clock ensemble time scale. Simulation verification was achieved for virtual clock ensembles. Preliminary results showed that the proposed technique could reduce the frequency instability of the clock ensemble time scale. Considering actual applications, we will optimize further the proposed method by considering the impact of choosing different combinations of clock subsets, reference clocks, numbers of subsets, and other parameters. It could be applied to a new high-performance time scale generated by the real clock ensemble in the next work.

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